On the theoretical understanding of Bayesian methods in complex models

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Bayesian methods are becoming increasingly popular in various fields of sciences. They offer a principled way to incorporate expert knowledge into the statistical model and provide built in uncertainty quantification (i.e quantifying the remaining uncertainty in the statistical procedure). To deal with the ever increasing amount of available information and increasingly more complex models new approximation methods (e.g. parallel computing, variational approximations) are being developed to speed up the computations and reduce the memory requirement. I will demonstrate the wide applicability of Bayesian methods over several concrete examples ranging from epidemiology through astronomy.

However, Bayesian methods are subjective by nature (by the choice of the prior) and inaccurate use can result in misleading interpretation and contradictory conclusions.
To better understand their behaviour and achieve acceptance by a wider scientific community their frequentist properties have to be understood. The main focus of research is on understanding whether Bayesian methods can recover the underlying parameters of interest (with an optimal rate) as the sample size increases (called Bayesian consistency), and whether Bayesian uncertainty quantification provides reliable confidence statement (i.e. frequentist coverage). I will briefly discuss the state of the art literature in this field and introduce some standard techniques achieving such guarantees.

References

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